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A study of three-dimensional natural convection in high-pressure mercury lamps—III. Arc centering by magnetic field

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Abstract—The present work constitutes the third part of a series of work with the goal of developing a three-dimensional computational model to help understand the various physical mechanisms involved in high-pressure discharge arcs. The effect of an externally applied magnetic field on the convection and temperature characteristics is studied in the present effort. The classic experiment of Kenty is first simulated. It is found that the magnetic field can induce a downward velocity to the hot gas core to offset the buoyancy-induced nonuniformity in the temperature field. Separate pairs of contra-rotating recirculating eddies resulting from the magnetic and buoyancy effects can be identified. Qualitative as well as quantitative agreements between Kenty's experiment and the present prediction are observed. The feasibility of applying the magnetic control to optimize the operation of the modern discharge lamp is then investigated. The model reveals that convection strengthens if either the pressure inside the arc-tube or the magnitude of the magnetic field increases. Furthermore, with the same geometry, the required strength of the magnetic field to center the hot core and maintain a symmetric temperature distribution is linearly proportional to the arc-tube pressure. This finding yields a very useful method for optimizing the discharge lamp design.

1. INTRODUCTION

THE HIGH-pressure high-temperature electric arc confined in cylindrical quartz tubes is an important device of many practical applications, notably for the modern high intensity lamp and laser. It has received much research attention due to the complexity of its intrinsic physics and chemistry [1–3]. To successfully develop a theoretical model to predict the characteristics of the arc, due attention must be paid to account for the aspects of energy balance in a detailed manner, including conduction, convection, radiation, ohmic heating, and any other energy transfer modes present. The highly coupled and nonlinear set of transport equations of mass continuity, linear momentum, and energy of the gas phase must be solved simultaneously in a geometry complicated by the existence of electrodes at both ends of the arc-tube and tube curvature. Among the many important mechanisms, natural convection plays a pivotal role in determining the qualitative as well as quantitative distribution of the temperature field in the arc-tube. In a classic study of convection in high-pressure Hg arcs, Kenty [1] has measured the convection velocity in a horizontal arc-tube, where natural convection buoys the arc upward, and reported the conditions under which an externally applied magnetic field straightens the arc.

Zollweg [4] and Lowke [5] were the first to conduct analytical and computational studies of natural convection in high-pressure mercury arcs. In their works, the fluid flow and energy equations in an axisymmetric domain are solved. Recently, progress has been made along the line of computational modeling in a three-dimensional domain [6–8], with the capability of handling the irregular and complex geometry of the arc-tube, including the curved tube surface and the electrode intrusion at the ends of the arc-tube. The model advanced in refs. [6–8] provides a solution to the combined momentum, mass, continuity, energy, and electric field equations, based on the fundamental conservation laws and simplified radiation treatment. These equations are solved using the finite volume algorithm in a generalized curvilinear coordinate system. Both straight and curved arc-tubes have been studied and their impact on overall performance predicted. Good agreements have been obtained between theoretical prediction and experimental measurement, in terms of mounting angle [6, 8], curvature effect [7], and wall temperature distribution [8]. Those favorable agreements indicate that the computational model based on the first principles can make a truly useful contribution for advancing our understanding of the complex physics present in the system.

As stated in Parts I and II [7, 8], the degree of uniformity of temperature distribution, both within the arc-tube and on the tube wall, can greatly affect the light quality and the life expectancy of the lamp. Also, it has been demonstrated that the temperature uniformity within the arc-tube can be achieved by modifying the tube curvature. This concept has in fact been put to use for real products. However, in order to continually supply new ideas to product design, it is desirable to identify other alternatives to attain this same goal. Here the model is extended to explore another promising thought, that is, the application of a magnetic field to control the convection and thermal characteristics in the arc-tubes. The tube geometry and the operating condition of Kenty [1] will first be simulated and the model prediction will be compared to the data reported in ref. [1]. Then the base geometry and condition employed previously in Parts I and II of this work [7, 8] will be adopted to explore the feasibility of applying the magnetic field to offset the buoyancy effect in order to straighten the arc in the modern discharge lamps. It is noted that, compared to Kenty's experiment, the modern discharge arc is confined within much smaller tubes but yields much higher energy intensity levels. The possible use of a magnetic field to control the natural convection effect is interesting not only from the viewpoint of basic scientific exploration but also the possibility of future technical innovation.

2. PROBLEM FORMULATION

A schematic illustration of the system set-up taken from the original work of Kenty [1] is shown in Fig. 1. Kenty observed that without a magnetic field, an arc in a straight tube rests against the top wall. This feature is well known and has already been reported in detail in ref. [7]. The focus here is to model Kenty's idea of using an electromagnet to straighten the arc and center the high temperature gas core in a horizontal arc-tube. As illustrated in Fig. 2, also taken from Kenty's paper, the flow pattern in the arc evolves as a function of magnetic field.

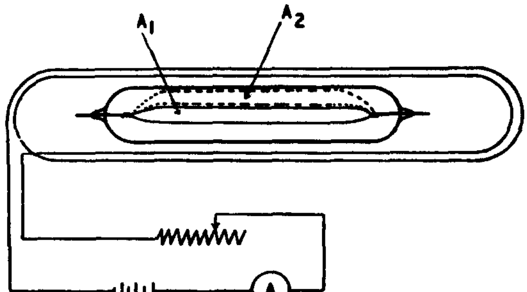

FIG. 1. Apparatus for measuring force necessary to keep horizontal arc in axial position (A_1). Without magnetic field arc rests against top wall (position A_2). Taken from Kenty [1].

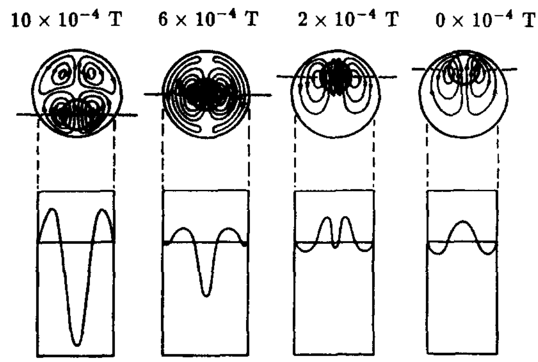

FIG. 2. Schematic view of convection currents in 400 W lamp operating horizontally under magnetic control. Four different arc positions are shown corresponding to four different field strengths. Below, also schematically, are the vertical components of convection speed at the levels indicated. Taken from Kenty [1].

The arc is assumed to be governed by the Navier–Stokes equations coupled with an equation for the flow of electric current in the arc fluid. The coupled equations can all be written in the strong conservation law form in Cartesian coordinates as follows:

$$\frac{\partial}{\partial x}(\rho u\phi)+\frac{\partial}{\partial y}(\rho v\phi)+\frac{\partial}{\partial z}(\rho w\phi)=\frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\Gamma\frac{\partial\phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\Gamma\frac{\partial\phi}{\partial z}\right)+R(x,y,z). \quad (1)$$

Here x, y, z, u, v and w are the usual Cartesian coordinates and fluid velocity components. The generalized density, ρ, potential, ϕ, diffusion coefficient, Γ, and source term, R, are defined in Table 1 for each of the governing equations. In the model, these equations are transformed to a generalized curvilinear coordinate system, and solved using a finite volume scheme. Grids included $29\times17\times17$ nodes for Kenty's tube and $29\times21\times23$ for the modern tube are considered.

A note should be made regarding the Lorentz force ($\mathbf{J}\times\mathbf{B}$) term presented in the momentum equations. Although the Lorentz force is applied predominantly to the electrons, it appears in our momentum equations as applied to the gas as a whole. As noted by McLain and Zollweg [9], if the momentum exchange between electrons and neutrons and ions is slow, then there could be a directed flow of charged particles carrying kinetic and potential energy away from the arc core that is not accounted for by the convection terms. However, McLain and Zollweg estimated that this portion of energy transfer is extremely small and hence can be safely neglected.

Boundary conditions are imposed on the surfaces which contain the discharge fluid—the surfaces of the arc-tube and the electrodes. For the energy equation, which determines the fluid temperature, T, a temperature of 1715 K is prescribed on the tips of the electrodes for Kenty's tube and 2400 K is prescribed

Table 1. Discharge equation terms

Equations	ρ	ϕ	Γ	$R(x, y, z)$
Energy	ρ	T	$\frac{k}{C_p}$	$\frac{\sigma(\nabla V \cdot \nabla V) - u_{rad}}{C_p}$
x, y, z momentum	ρ	$\mathbf{v}$	μ	$-\nabla p + \rho \mathbf{g} - \mathbf{J} \times \mathbf{B}$
Continuity	ρ	1	0	0
Electrostatic potential	0	V	σ	0

Table 2. Fluid property coefficients in SI units

Property	A	B	C	D	E
σ			0.56×10^7	65 200	
k	0.626×10^{-2}	0.122×10^{-4}	0.176×10^{11}	159 000	
u_{rad}	-0.466×10^6		0.971×10^{15}	95 500	
μ	0.299×10^{-4}	0.672×10^{-7}			
ρ					0.351×10^4
C_p	0.116×10^3				

for the modern tube, respectively. The tube wall temperature is iteratively adjusted during the course of calculation to satisfy a heat flux balance for the quartz arc-tube wall. The total power consumption modeled for both Kenty's and our reference cases is 400 W. For the three momentum equations, which determine the fluid velocity, $\mathbf{v}$, a no-slip condition is imposed at the surfaces. A Laplace equation determines the electrostatic potential, V, where the electrical conductivity, σ, is a strong function of temperature. For the Laplace equation, the boundary conditions involve treating the electrode surfaces as conducting, and the quartz walls as insulating. Thus, the electric current, $\mathbf{J} = \sigma \nabla V$, flows through the fluid from one electrode to the other. The potential difference between the electrodes is adjusted to hold the total ohmic power dissipated in the discharge fluid at a preset level.

Relative to Parts I and II, the extension to the model introduced here includes the effect of an externally applied magnetic field. The magnetic field enters simply as the $\mathbf{J} \times \mathbf{B}$ source term in the momentum equations.

Fluid properties, assumed to be functions solely of temperature, are required as inputs to the model. These properties are determined with a separate model which provides a detailed treatment of chemistry and radiation trapping for a vertical axisymmetric arc assumed to exhibit local thermodynamic equilibrium [10]. The fluid properties from this separate model are fitted to a function of the form

$$f(T) = A + BT + C\exp(-D/T) + E/T. \quad (2)$$

The fluid property coefficients determined for Kenty's conditions are shown in SI units in Table 2.

3. RESULTS AND DISCUSSION

Kenty's experiment [1] will be first compared to assess and validate the numerical prediction. Next,

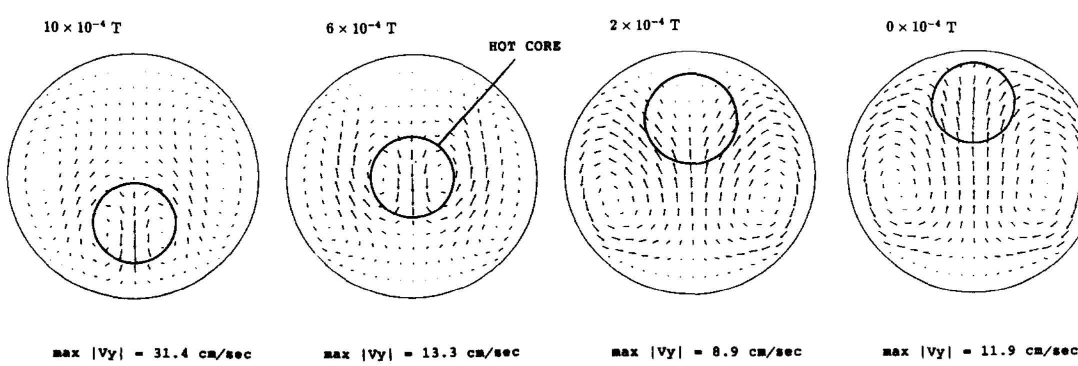

FIG. 3. Predicted convection patterns and hot core locations (in mid-section) under magnetic control corresponding to four different field strengths.

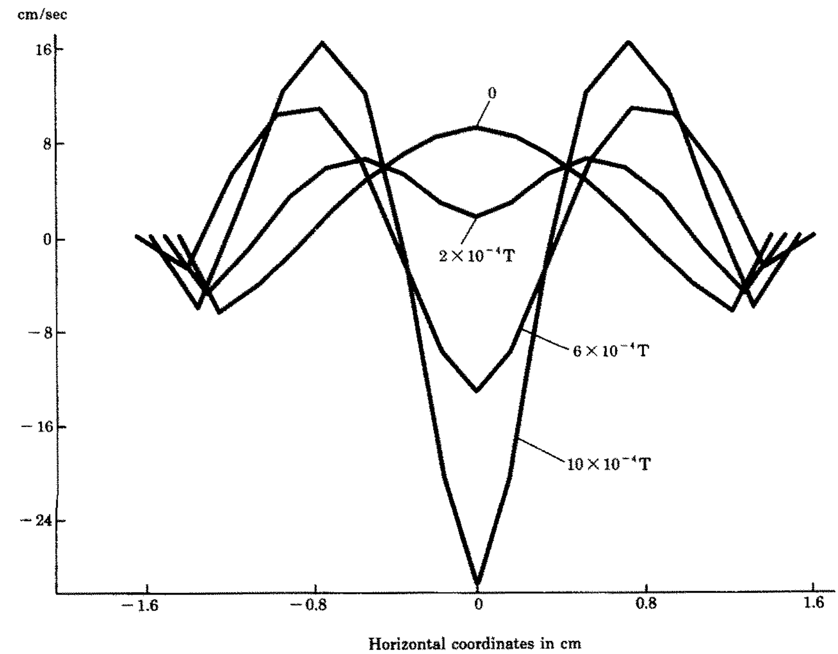

FIG. 4. Predicted vertical velocity component profiles plotted along the horizontal line passing through the hot core (in mid-section). Each curve corresponds to a different field strength but same tube pressure.

some design-related questions, mainly the effect of the tube pressure on the interaction between natural convection and magnetic field, will be investigated.

In Kenty's study, the quartz arc-tube has an inner diameter of 3.33 cm, an electrode separation of 15.5 cm, and a Hg loading of 11.5 mg cm^{-1}, which results in a Hg pressure of about 1 atm. The power rating of the arc-tube is 400 W for the cases to be compared. Kenty also injected tracer particles of CaO and MgO into the arc fluid to enable direct visualization of the convection characteristics. A schematic view of the convection pattern in a 400 W straight arc-tube operating horizontally under magnetic control observed by Kenty is reproduced in Fig. 2 to aid our discussion and model validation. Four different arc positions are shown corresponding to four different magnetic field strengths. Kenty also noted that the scale in Fig. 2 is such as to make the maximum speed shown about 20 or 30 cm s^{-1}.

The measured magnetic forces in the four positions in Fig. 2 were 2.5, 1.5, 0.5 and 0 dynes cm^{-1}, corresponding to magnetic strengths of, respectively, 10×10^{-4}, 6×10^{-4}, 2×10^{-4}, and 0 T. With no magnetic control, the well-known convection pattern of a pair of contra-rotating cells is observed both experimentally by Kenty and numerically by the present model [7]. With increasing effect of the magnetic force, the convection field progressively exhibits more complicated characteristics. The most noticeable feature is that the particles in the middle of the arc-tube are convected along the downward direction, i.e. a reversal of convection direction results from the imposed magnetic field. However, as illustrated in Fig. 2, the original pair of contra-rotating convection cells induced by the buoyancy effect is still present, albeit at different locations and with different convection strengths. As the magnetic force becomes stronger, the overall convection field strengthens, inducing a higher downward velocity at the center of the arc, and causing the recirculating eddies created by the buoyancy and magnetic effects to separate more clearly from each other.

The numerical solutions yielded by the present model with conditions corresponding to Kenty's experiment are shown in Figs. 3 and 4. In Fig. 3, the predicted convection pattern along with the position of the hot gas core are shown. In Fig. 4, the vertical velocity component profiles along the horizontal line passing through the center of the hot core are compared. The overall agreements between Figs. 2–4 in terms of the evolution of the convection pattern, the location of the hot gas core, and the velocity profile with respect to the strength of the magnetic force are good. Most importantly, it is confirmed both experimentally and theoretically that an externally imposed magnetic field is an effective way to straighten and center the arc. To understand the physics involved, it should first be noted that the primary impact of the magnetic field is to interact with the fluids carrying electric current. Since the hot gas core is the region that the electric conductivity, and hence the electric current, is strongest (as shown by Table 2), the downward velocity caused by the magnetic field is highest there. Hence, the magnetic field exerts a direct influence on the fluid motion only in regions where the temperature is high. In regions close to the tube wall, natural convection dominates the transport process since it depends on the gradient, not the absolute level, of the temperature field. By recognizing the individual effect of the magnetic field and the buoyancy, one can devise a direct control strategy to adjust the position of the hot core and to improve the symmetry of the temperature field. It is also noted that the velocity magnitudes predicted by the model are in the comparable quantitative range reported by Kenty.

Next, our attention is turned to investigate the issues of applying the concept of magnetic control to the modern discharge arc-tube. The objective is to identify the strength of the magnetic field required for optimizing the temperature distribution within the tube. The base geometry and the operating conditions are identical to those given in Table 1 of Part I [7].

Compared to Kenty's arc-tube, the modern design is substantially smaller in length and diameter. However, the tube pressure is considerably higher in the modern lamp, all indicating the progress made during the last five decades in lighting technology. Figure 3 of Part I shows that with a tube pressure of 4 atm, a substantial buoyancy effect results from the intense heating process and, consequently, the arc is noticeably lifted towards the top wall. It is found that with a magnetic field of 3×10^{-4} T, the hot gas core can be satisfactorily repositioned to the center of the arc-tube, producing a highly symmetric temperature distribution. The temperature as well as the convection fields of the solution under the influence of the magnetic field are shown in Fig. 5. Compared to Fig. 3 of Part I, the cross-section is now occupied by the recirculating eddies induced by the magnetic field, which is similar to Kenty's case. The original pair of eddies induced by the buoyancy effect has been pushed toward the region very close to the side wall.

Figure 6 shows that by varying the pressure inside the same arc-tube, the hot core remains centered as long as the magnetic strength is appropriately adjusted. Furthermore, it appears that there exists a very simple linear relationship between the tube pressure and the required magnetic strength. However, as demonstrated in Fig. 7, where the vertical velocity profiles along the horizontal line passing through the hot core, which coincides with the geometrical center region, are compared, the overall convection strength with optimized arc location increases monotonically as the operating pressure, and hence the magnetic strength, increases. On the other hand, the buoyancy-induced recirculating cells, now weak and confined to the wall region, maintain the same convection strengths for all cases.

4. SUMMARY AND CONCLUSION

Figure 8 summarizes the implications of the design options considered in Parts I, II and the present work. Originally, with a straight arc-tube placed horizontally, the buoyancy effect creates an upward bowing arc and a nonuniform temperature distribution. A pair of contra-rotating recirculating eddies is observed. With 4 atm tube pressure and the geometrical information listed in Table 1 of Part I, the overall convection strength is about 15 cm s^{-1} for the base design shown in Fig. 8(a). As demonstrated in Part II, a bow-shaped arc-tube can adjust the temperature field to create a symmetric distribution. The final optimized design and resulting thermal and convection characteristics of the bow-shaped arc-tube are shown in Fig. 8(b). This type of design can be viewed as a passive thermal control in that a symmetric temperature distribution is achieved by 'accommodating' the intrinsic buoyancy-induced transport through

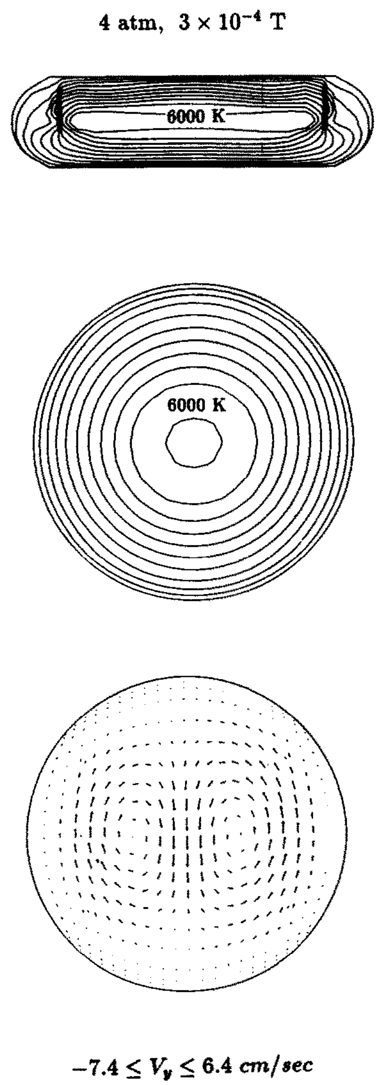

FIG. 5. Predicted temperature contours and velocity field in central plane and mid-section. The most inner temperature contours are 6000 K and contour intervals are 500 K.

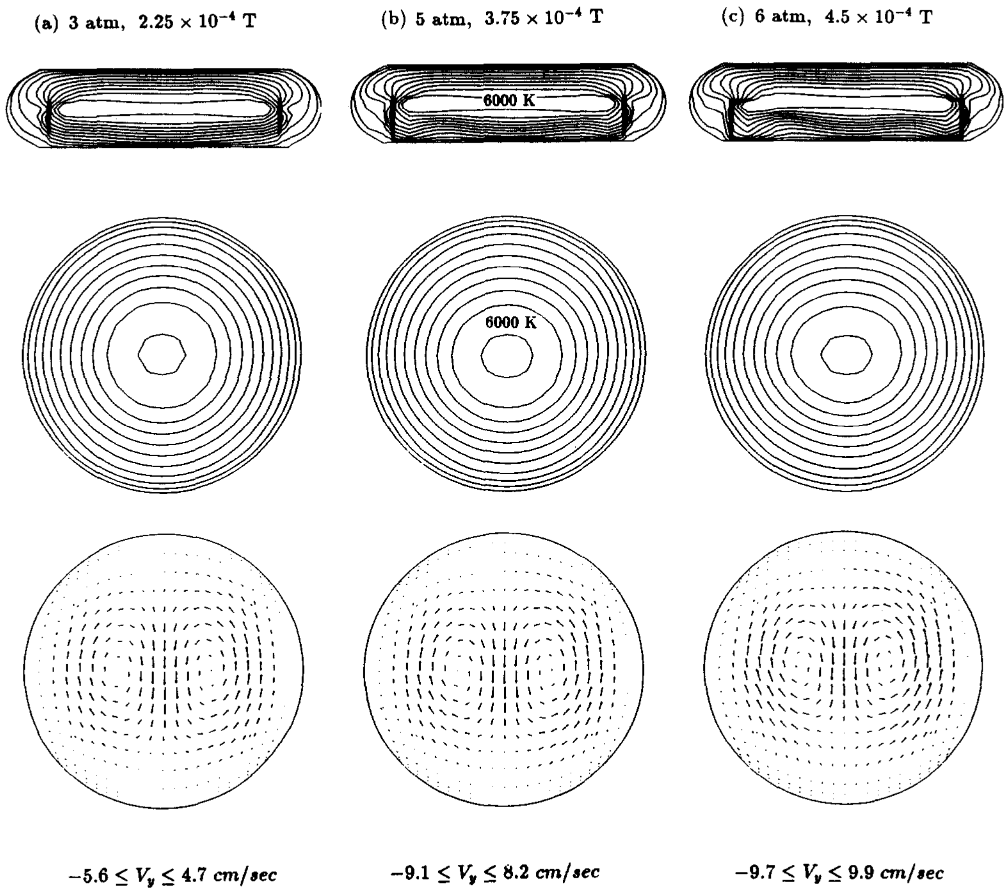

FIG. 6. Predicted temperature contours and velocity fields in central plane and mid-section. The most inner temperature contours are 6000 K and contour intervals are 500 K. Each case corresponds to a given tube pressure and a magnetic field strength.

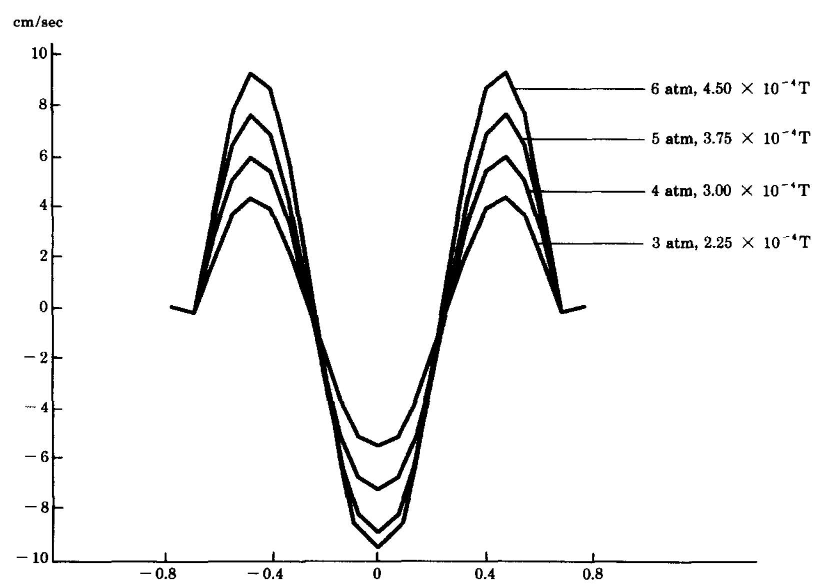

FIG. 7. Predicted vertical velocity component profile plotted along the horizontal line passing through the hot core (in mid-section) which coincides with the geometrical center for these cases. Each curve corresponds to a given tube pressure and magnetic field strength.

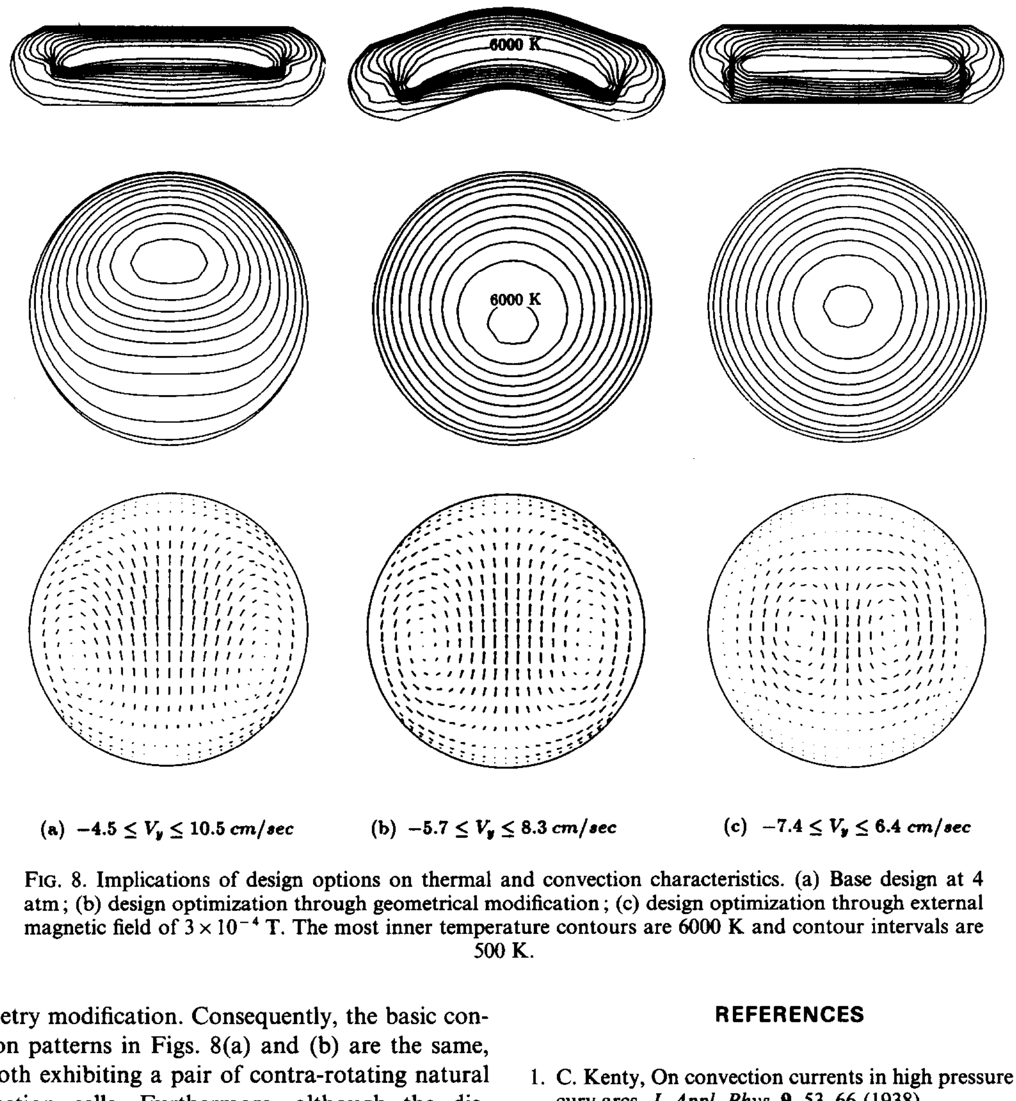

FIG. 8. Implications of design options on thermal and convection characteristics. (a) Base design at 4 atm; (b) design optimization through geometrical modification; (c) design optimization through external magnetic field of 3×10^{-4} T. The most inner temperature contours are 6000 K and contour intervals are 500 K.

geometry modification. Consequently, the basic convection patterns in Figs. 8(a) and (b) are the same, i.e. both exhibiting a pair of contra-rotating natural convection cells. Furthermore, although the distributions of the velocity field are different in detail, the overall convection strength remains essentially the same. The application of an externally imposed magnetic field can be viewed as an active control strategy since it seeks to change the intrinsic physical mechanisms involved. By inducing a downward velocity to the fluid particles carrying electric current, i.e. those with high temperature levels, the position of the hot gas core can be adjusted according to the strength of the magnetic field. The convection field now responds to both buoyancy and magnetic fields, resulting in multiple pairs of cells. As shown in Fig. 8(c), with an applied magnetic strength of 3×10^{-4} T, a symmetric temperature distribution can be produced. It is interesting to observe that even though the recirculation directions of the major convection cells are different, the overall convection strength maintained in Fig. 8(c) still remains about the same.

It is concluded that two effective means of design optimizations have been successfully identified for the high-pressure discharge arc-tube. Both methods should find their areas of application.

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UNE ETUDE DE LA CONVECTION NATURELLE TRIDIMENSIONNELLE DANS LES LAMPES A HAUTE PRESSION DE MERCURE—III. CENTRAGE DE L'AXE PAR UN CHAMP MAGNETIQUE

Résumé—Cette étude constitue la troisième partie d'une étude dont le but est le développement d'un modèle tridimensionnel de calcul pour aider la compréhension des différents mécanismes physiques en jeu dans la décharge d'arcs à haute pression. On traite ici de l'effet d'un champ magnétique externe sur la convection et la température. L'expérience classique de Kenty est simulée. On trouve que le champ magnétique peut induire une vitesse descendante du gaz chaud et imprimer une hétérogénéité, due au flottement, dans la champ de température. On peut identifier des paires séparées de tourbillons contrarotatifs résultant des effets du magnétisme et du flottement. On observe des accords qualitatifs et quantitatifs entre l'expérience de Kenty et les calculs. On explore la faisabilité d'un contrôle magnétique pour optimiser le fonctionnement d'une lampe moderne à décharge. Le modèle révèle que la convection se renforce si on augmente la pression ou l'importance du champ magnétique. Avec la même géometrie, l'intensité requise du champ magnétique au centre du coeur chaud pour maintenir une distribution symétrique de température est directement proportionnelle à la pression du tube à arc. Ceci conduit à une manière pratique pour optimiser le dessin des lampes à décharge.

UNTERSUCHUNG DER DREIDIMENSIONALEN NATÜRLICHEN KONVEKTION IN HOCHDRUCK-QUECKSILBERLAMPEN—III. ZENTRIERUNG DES LICHTBOGENS DURCH EIN MAGNETFELD

Zusammenfassung—Die vorliegende Arbeit stellt den dritten Teil einer Reihe von Arbeiten dar, deren Ziel die Entwicklung eines dreidimensionalen Rechenmodells ist, welches zum Verständnis der verschiedenen physikalischen Mechanismen bei der Hochdruck-Lichtbogenentladung beitragen soll. In diesem Teil der Arbeit wird der Einfluß eines von außen angelegten Magnetfeldes auf die Konvektion und die Temperatur-Kenngrößen untersucht. Zuerst wird das klassische Experiment von Kenty simuliert. Es zeigt sich, daß das magnetische Feld eine Abwärtsströmung des heißen Gaskerns erzeugen kann, welche die durch Auftriebskräfte hervorgerufenen Ungleichförmigkeiten des Temperaturfeldes kompensiert. Es wurden getrennte Paare von gegensinnig rotierenden Rezirkulationswirbeln identifiziert, die durch die magnetischen und die Auftriebskräfte verursacht werden. Die vorgelegten Berechnungen stimmen qualitativ und quantitativ mit Kentys Versuchsergebnissen überein. Anschließend wurde die Anwendbarkeit der magnetischen Beeinflussung für die Optimierung moderner Bogenentladungslampen untersucht. Das Modell ergab, daß die Konvektion stärker wird, wenn entweder der Druck innerhalb des Bogenrohrs oder die Stärke des magnetischen Feldes zunimmt. Zudem ist die Stärke des magnetischen Feldes, die für eine Zentrierung des heißen Kerns und für eine symmetrische Temperaturverteilung erforderlich ist, direkt proportional zum Druck. Die vorgestellte Methode ist somit für die Optimierung von Bogenlampen sehr nützlich

ИССЛЕДОВАНИЕ ТРЕХМЕРНОЙ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ В РТУТНЫХ ЛАМПАХ С ВЫСОКИМ ДАВЛЕНИЕМ. Ш. ЦЕНТРИРОВАНИЕ ДУГИ ЗА СЧЕТ МАГНИТНОГО ПОЛЯ

Аннотация—Данная работа является третьей частью исследований, предпринятых с целью разработки трехмерной вычислительной модели, способствующей пониманию различных физических механизмов дуговых разрядов при высоких давлениях. Определяется влияние внешнего магнитного поля на характеристики процесса. В первую очередь моделируется классический эксперимент Кенти. Найдено, что под воздействием магнитного поля неоднородность температурного поля может компенсироваться за счет скорости течения в направлении к нагретому ядру газа. Можно выявить отдельные пары вращающихся в противоположных направлениях рециркулирующих вихрей, образующихся за счет эффектов магнитного поля и подъемных сил. Наблюдается качественное и количественное согласие между экспериментальными данными, полученными Кенти, и результатами настоящего исследования. Рассматривается возможность использования магнитного регулирования для оптимизации работы современной разрядной лампы. Модель показывает, что конвекция усиливается с ростом давления внутри дуговой трубка или величины магнитного поля. Найдено также, что при той же геометрии напряженность магнитного поля, необходимая для центрирования нагретого ядра и поддержания симметричного распределения температур, пропорциональна давлению в дуговой трубке. Таким образом, в работе предложен эффективный метод оптимизации конструкции разрядной лампы.